Columbia Business School

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B9824: Foundations of Optimization, Fall 2009

Course Description

Mathematical optimization provides a unifying framework for studying issues of rational decision-making, optimal design, effective resource allocation and economic efficiency. It is therefore a central methodology of many business-related disciplines, including operations research, marketing, accounting, economics, game theory and finance. In many of these disciplines, a solid background in optimization theory is essential for doing research.

This course provides a rigorous introduction to the fundamental theory of optimization. It examines optimization theory in deterministic settings, including optimization in $\mathbb{R}^n$ and as well as in more general vector spaces. The course emphasizes the unifying themes (optimality conditions, Lagrange multipliers, convexity, duality) that are common to all these areas of mathematical optimization. Applications across a range of problem areas also play a key role in the class. The goal of the course is to provide students with a foundation sufficient to use basic optimization in their own research work and/or to pursue more specialized studies involving optimization theory.

The course is open to all students, but it is designed for entering doctoral students. The prerequisites are calculus, linear algebra and some familiarity with real analysis. Other concepts (e.g., vector spaces) are developed as needed throughout the course.

Course Outline

1. Introduction
   (a) Basic problems and motivating examples
   (b) Review of real analysis and linear algebra
2. Local theory of optimization
   (a) Unconstrained optimization: Weierstrass' Theorem, first- and second-order conditions, gradient methods
   (b) Constrained optimization: Lagrangian, KKT conditions
3. Global theory of optimization
   (a) Convex sets and functions, implications of convexity for optimization
   (b) Duality: geometric interpretation, strong and weak duality, properties of dual problems, duality for LPs and QPs, conjugate duality

4. Applications: Approximation and fitting, estimation, geometric problems

5. Vector space methods: vector spaces, inner products and norms, the projection theorem, dual spaces, generalized KKT conditions

Required Texts

- D. P. Bertsekas, *Nonlinear Programming*, 2nd Edition. Athena Scientific, 1999.
- S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004. Available online at `http://www.stanford.edu/~boyd/cvxbook`.
- D. G. Luenberger, *Optimization by Vector Space Methods.* Wiley, 1969.

Selected References

Real Analysis:

- W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition. McGraw-Hill, 1976.

Linear Algebra:

- G. Strang, *Linear Algebra and Its Applications*, 3rd Edition. Brooks Cole, 1988.

Optimization:

- D. P. Bertsekas, *Convex Optimization Theory.* Athena Scientific, 2009.
- D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, 3rd Edition. Springer, 2008.

Coursework and Grading

Several homework assignments will be given out during the semester. There will be a final exam. The course grade will be the weighted average of the homework (50%) and the final (50%).

Office Hours

I am generally available in my office (Uris 416) Monday–Friday during the day. You are welcome to stop by without notice if you have short questions. If you have more involved questions or need extensive help, it would be best if you emailed me to make an appointment.