Columbia Business School

Professor Ciamac Moallemi
email: ciamac@gsb.columbia.edu

B9824: Foundations of Optimization, Fall 2008

Course Description

Mathematical optimization provides a unifying framework for studying issues of rational decision-making, optimal design, effective resource allocation and economic efficiency. It is therefore a central methodology of many business-related disciplines, including operations research, marketing, accounting, economics, game theory and finance. In many of these disciplines, a solid background in optimization theory is essential for doing research.

This course provides a rigorous introduction to the fundamental theory of optimization. It examines optimization theory in deterministic settings, including optimization in $\mathbb{R}^n$ and optimization over time (optimal control). The course emphasizes the unifying themes (optimality conditions, Lagrange multipliers, convexity, duality) that are common to all these areas of mathematical optimization. Applications from problem areas in which optimization plays a key role are also introduced. The goal of the course is to provide students with a foundation sufficient to use basic optimization in their own research work and/or to pursue more specialized studies involving optimization theory.

The course is open to all students, but it is designed for entering doctoral students. The prerequisites are calculus, linear algebra and some familiarity with real analysis. Other concepts (e.g., vector spaces) are developed as needed throughout the course.

Course Outline

1. Introduction: basic problems and motivating examples
2. Review of real analysis and linear algebra
3. Local theory of optimization
   (a) Unconstrained optimization: Weierstrass' Theorem, first- and second-order conditions, gradient methods
   (b) Constrained optimization: Lagrangian, KKT conditions
4. Global theory of optimization
   (a) Convex sets and functions, implications of convexity for optimization
   (b) Duality: geometric interpretation, strong and weak duality, properties of dual problems, duality for LPs and QPs, conjugate duality
   (c) Applications

5. Dynamic optimization
   (a) Calculus of variations
   (b) Optimal control: general formulation, first- and second-order conditions, bang-bang control, Pontryagin's Maximum Principle
6. Vector space methods: vector spaces, inner products and norms, the projection theorem, dual spaces, generalized KKT conditions

Required Texts

- D. G. Luenberger, *Optimization by Vector Space Methods.* Wiley, 1969.
- D. P. Bertsekas, *Dynamic Programming and Optimal Control, Volume 1*, 3rd Edition. Athena Scientific, 2005.

Selected References

Real Analysis:

- W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition. McGraw-Hill, 1976.

Linear Algebra:

- G. Strang, *Linear Algebra and Its Applications*, 3rd Edition. Brooks Cole, 1988.

Optimization:

- D. P. Bertsekas, *Nonlinear Programming*, 2nd Edition. Athena Scientific, 1999.
- S. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004. Available online at `http://www.stanford.edu/~boyd/cvxbook`.

Coursework and Grading

Several homework assignments will be given out during the semester. There will be a midterm exam and a final exam. The course grade will be the weighted average of the homework (10%), the midterm (30%), and the final (60%).

Office Hours

I am generally available in my office (Uris 416) Monday–Friday during the day. You are welcome to stop by without notice if you have short questions. If you have involved questions or need extensive help, it would be best if you emailed me to make an appointment.