Quantifying the Value of Revert Protection

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Abstract. Revert protection is a feature provided by some blockchain platforms that prevents users from incurring fees for failed transactions. This paper explores the economic implications and benefits of revert protection, in the context of priority auctions and maximal extractable value (MEV). We develop an equilibrium game theoretic model that captures the behavior of users (MEV searchers) bidding to have their transaction included ahead of others, in an environment where only a single transaction will succeed in realizing the common value of an opportunity, and in settings both with and without revert protection. Our model applies to a broad range of settings, including Layer 1 (L1) blockchains (e.g., Ethereum mainnet) and Layer 2 (L2) blockchains, and auctions such as "bundle auctions" (on L1s) or priority ordering auctions (on L2s). We establish that, in the absence of revert protection, users will employ randomized strategies to mitigate the impact of paying for failed transactions. This will ultimately result in less auction revenue, despite the fact that failed transactions still pay fees. Our results quantify in closed form how revert protection enhances auction revenue, and also improves market efficiency and provides for more efficient use of blockspace, as a function of the underlying parameters (the value of the MEV opportunity, the base fee, the revert penalties, and the number of participating agents).

Keywords: Blockchain · Priority gas auction · Revert protection · MEV

1 Introduction and Background

Revert protection is a feature for blockchains that block builders can provide where they exclude transactions that would otherwise fail, protecting users from paying fees for failed transactions. However, failed transactions would pay fees, which often accrue to the block builder. So is it in the interest of those builders to offer revert protection?

In this paper, we make the case that it is, and quantify by how much. We study the implications of revert protection on blockchain platforms, and particularly on the auctions that many of those platforms use to capture and allocate

maximal extractable value (MEV). We show that in many relevant settings, revert protection is beneficial for auctioneer revenue, as well as for other relevant outcomes like price discovery and blockspace efficiency.

The analysis is complicated by the wide variety of rules $-$ across a diverse variety of L2s, as well as in the complex Ethereum block builder market for how transactions are selected for inclusion and ordered, how fees are collected, and who those fees go to, as well as how applications behave under those rules. We propose a model that can be parameterized to cover a wide variety of these settings, and use that model to solve for the equilibrium behavior of MEV searchers as a function of those parameters.

1.1 Revert Protection

On Ethereum and similar blockchains, users interact with smart contracts using atomic transactions. During execution of a transaction, one of those smart contracts may trigger a "revert," which causes the entire transaction to fail. Normally, if the transaction is included, the user who sent the reverting transaction is still charged some transaction fee, even though the transaction has no other effect on the blockchain state.

The fee charged is typically the product of the transaction's gas price and the gas used by the transaction before it either completes or reverts. The gas price, in turn, can be separated into the "base fee"—which can be thought of as a flat fee that must be paid by any transaction in the block—and the "priority fee"— an additional fee that is paid to the builder and often affects ordering as well as inclusion (particularly when blocks are full).

The gas used in the transaction is a function of how much of the transaction was executed—a transaction that reverts will usually use only a portion of the gas that it would have used if it succeeded. For example, on an automated market maker, typical reverting transactions may use only around $10-20\%$ of the gas that a succesful transaction would use.

Some block builders implement a feature where they exclude failed transactions entirely, thus saving the user from having to pay a fee for it. This feature is known as "revert protection." This feature improves user experience, since users only have to pay a fee if their transaction succeeds. However, it also has significant effects on the behavior of the profit-seeking bots known as "searchers," which we now describe.

1.2 Searchers, Auctions, and Block Building

Public blockchains have abundant opportunities for MEV. For example, a major type of MEV is the arbitraging of prices between decentralized and centralized exchanges, also called "CEX-DEX arbitrage," as discussed in [\[2\]](#page-14-0) and [\[8\]](#page-14-1). On these blockchains, independent actors known as "searchers" seek out MEV opportunities and compete to fill them.

Some block builders use different types of auctions to allocate these opportunities to searchers. For a given MEV opportunity, each transaction trying to claim it can be thought of as a "bid." Generally, only one transaction trying to claim a given opportunity will succeed in extracting the value from it. Revert protection is therefore very relevant for these auctions, because it protects bidders on a given MEV opportunity from having to make a payment for failed bids. Two examples of block building auctions include the "bundle auctions" run by builders such as Flashbots on Ethereum mainnet, and "priority auctions" run by sequencers on Ethereum L2s.

Bundle Auctions. On Ethereum mainnet, blocks are typically built by profitmaximizing "builders."⁴ Many of these builders run "bundle auctions" in which they allow any searcher to submit transactions, and use those searchers' bids as a factor when deciding which transactions to include and how to order them.

There are two ways bids can be expressed in this auction. First, through the priority fee on the transaction. Second, by making a payment directly as part of the logic of the transaction (by calling coinbase.transfer (\ldots)), as discussed in [\[3\]](#page-14-2). The relevant difference between these methods of payment is that (some portion of) priority fees are paid even if the transaction reverts, whereas if the transfer is made as part of the transaction, then it will be conditional on whether the transaction succeeds.

On Ethereum mainnet, the different components of gas price — base and priority fees — are paid in ETH and accrue to different users. Base fees are burned (meaning they ultimately accrue to all ETH holders, rather than the builder). Priority fees accrue to the block builder.⁵

Many builders, such as Flashbots [\[5\]](#page-14-3), provide revert protection for transactions submitted to them, even though it is not required by the Ethereum protocol and they might receive more in transaction fees by including it. Our results in this paper help explain this choice by showing that it likely increases their expected revenue in equilibrium.

Priority ordering auctions on L2. On Layer 2 blockchains today, blocks are typically built by a single sequencer, which often follows a deterministic algorithm for transaction inclusion and ordering. One of the most popular algorithms for this is priority ordering, in which transactions are ordered in descending order of their gas price.

Priority ordering can be thought of as an auction in which transactions bid with the discretionary part of their gas price (i.e., their priority fee) to be in-

 4 Block builders also generally play the role of bidders themselves in the "MEV-Boost" $\,$ auction, in which they bid to have their block included by the current proposer, as discussed in [\[4\]](#page-14-4). Since in that auction, "bids" are complete blocks, revert protection is not as relevant for it, and we will mostly set it aside for purposes of this paper. Its implications for the bundle auction are noted in footnotes below.

 5 Note that these bids — whether paid through the priority fee or through a transfer to the block's coinbase address, technically go to the block proposer, not the block builder who is assembling the block. However, since they reduce the amount that the builder has to pay to the proposer to win the MEV-Boost auction, we can think of these payments as a value transfer to the builder.

cluded earlier in the chain. By default, this means that certain kinds of MEV (such as top-of-block CEX-DEX arbitrage) will generally accrue to the sequencer through priority fees. This means that when such transactions fail, they will still pay some of their priority fee to the sequencer. However, as discussed in [\[10\]](#page-14-5), there is a technique called MEV taxes that applications can use to capture all but a negligible portion of the value that would otherwise be paid through priority fees. Since MEV taxes are paid as part of the transaction and revert if the transaction fails, these fees will only be paid if the transaction succeeds.

Considering All Cases. Even just within these two settings of L1 block builders and L2 priority-ordered sequencers, we now need to consider at least seven cases that differ in who fees go to and how much is paid when the transaction reverts. Table 1 shows the differences between these cases.

Setting	Base fee goes to:	Rest of bid goes to:	Is base fee paid when TX fails?	Is rest of bid paid when TX fails?
L1 block builder with bids paid via priority fees	ETH holders	Builder	Partial	Partial
L1 block builder with bids paid via Coinbase transfer	ETH holders	Builder	Partial	N _o
L ₂ sequencer with priority ordering	Sequencer	Sequencer	Partial	Partial
L ₂ sequencer with priority ordering for apps using MEV taxes	Sequencer	Application	Partial	No
L1 block builder with revert protection	ETH holders	Builder	No	No
L ₂ sequencer with revert protection	Sequencer	Sequencer	No	No
L ₂ sequencer with revert protection for apps using MEV taxes	Sequencer	Application	No	No

Table 1. Transaction Fee Distribution and Revert Behavior

Our model is general enough to compute expected base fee and remaining fee (meaning priority fee, transfer to the coinbase address, or MEV tax) for all of these, by setting different parameters.

1.3 Execution Costs of Revert Protection

This paper is primarily concerned with the potential effects of revert protection, not with its implementation. We should note that feasible revert protection is a difficult technical challenge. Knowing whether a transaction will revert usually requires a builder to execute part of that transaction, which consumes some computational resources. If the builder does not charge the sender anything for that reverted transaction, it may be possible to denial-of-service (DOS) attack the builder with an overwhelming number of reverting transactions.

There are some mitigations to this kind of attack. In some settings, the builder may be able to use out-of-band spam prevention techniques (such as IP blocking) to make this DOS attack unfeasible. For certain special cases including the auction-like use cases considered in this paper—it may also be possible for the builder to determine statically whether a transaction will succeed or fail.

For purposes of this paper, we leave those challenges out of scope, and assume that—at least for the use cases discussed here—the builder is able to implement revert protection with negligible cost.

1.4 Our Contributions

The contributions of this paper are as follows:

- We introduce a novel, unified game theoretic model that can be used to analyze revert protection in a variety of settings such as L1 block builders, L₂ priority-ordered sequencers, MEV taxes, etc.
- We are able to solve for equilibria in our model in closed form in the model parameters: the value of the MEV opportunity, the base fee, the revert penalty parameters, and the number of participating agents.
- Using our equilibrium model, we can quantify the benefits of revert protection versus not offering revert protection:
	- Revert protection offers higher auction revenue. Both with and without revert protection, in our setting, the auctioneer can extracts all value when the auction clears. However, in the absence of revert protection, agents randomize when they participate, and there is a non-zero probability that the auction does not clear and value is lost. This results in reduced sequencer revenue.
	- In the context of automated market making, revert protection offers better market efficiency. Here, each auction represents a CEX-DEX arbitrage opportunity. In the absense of revert protection, there is some chance auctions do not clear, leaving arbitrage opportunities unexploited and hence prices less accurate.
	- Revert protection offers better block space efficiency. Indeed, when there is revert protection, only a single, winning transaction consumes block space. On the other hand, without revert protection, all submitted transactions consume block space.
- Our model allows different reversion penalty rates for the base fee and for priority fees. While the reversion penalty for priority fees influences bidder behavior, we establish that it does not influence aggregate system outcomes such as revenue, the probability that the auction clears, or the number of submitted transactions.

1.5 Literature Review

The dynamics of onchain MEV auctions were first explored in seminal work by Daian et al. [\[2\]](#page-14-0). Rasheed et al. mention the congestion effects of reverting transcations in [\[9\]](#page-14-6). Fox, Pai, and Resnick wrote about the feasibility of onchain auctions in [\[6\]](#page-14-7), as did Robinson and White in [\[10\]](#page-14-5). The whitepaper for Unichain, an L2 that plans to offer revert protection, referred briefly to its efficiency benefits for decentralized exchanges [\[1\]](#page-14-8).

Relative the economics literature on auctions, the model we consider is a common value, all pay auction, with a minimum allowable bid, and differential partial refunds for failed bids on the minimum bid and amounts above the minimum bid. Closest to our work in the paper of Hillman and Samet [\[7\]](#page-14-9). They consider a common value, all pay auction, with a minimum allowable bid, but do not allow for partial refunds of failed bids.

2 Model

2.1 Auction Description

We present a stylized model of a priority ordering auction for an MEV opportunity. There are $N \geq 2$ agents, indexed by $i \in [N]$, bidding for a single MEV opportunity in a block with common value $V > 0$, and the base gas fee for the block is $q > 0$.

Each agent i may choose to abstain or submit a bid ("priority gas fee") $b_i \geq 0$; we denote the action of abstaining by $b_i = \emptyset$. The winner, denoted by w, extracts value V from the MEV opportunity and pays $q + b_w$. In the event of a tie, the winner is randomly selected among the highest bids. Any losing agent j does not receive any value and incurs a revert cost of $r_1g + r_2b_j$, where $r_1, r_2 \in [0, 1]$ are the revert penalty rates on the base gas and priority gas fees, respectively.⁶

This model provides a unified setting that captures a variety of proposed and currently in-use block building protocols among popular blockchains, including those discussed in Section [1.2](#page-1-0) and listed in Table [1.](#page-3-0) Table [2](#page-6-0) illustrates how the revert penalty parameters may be set in various settings. For example, with an L1 block builder and bids paid via priority fee, we would expect $r_1 = r_2 > 0$, and these parameters might take a value of 10–20% for automated market maker swap transactions.

In the cases involving MEV taxes in Table [2,](#page-6-0) observe that $r_2 = 0$. This is because with MEV taxes, applications can decide what fraction of the priority fees to capture themselves versus giving to the sequencer. Priority fees that are captured by the application are fully refunded on revert. As we will see later on (cf. Theorem [4.1\)](#page-10-0), the total priority fee revenue does not depend on the choice

⁶ This model corresponds to a common value, all-pay auction with a minimum bid g and differential refunds $(1 - r_1)g$ and $(1 - r_2)b_j$ for the base bid amount g and additional bid amount b_j , respectively.

Setting	Revert penalty	Revert penalty	
	on base fee	on rest of bid	
L1 block builder with bids	$r_1 = r_2 \in (0,1]$		
paid via priority fees			
L1 block builder with bids		$r_2 = 0$	
paid via Coinbase transfer	$r_1 \in (0,1]$		
L ₂ sequencer with	$r_1 = r_2 \in (0,1]$		
priority ordering			
L ₂ sequencer with priority	$r_1 \in (0,1]$	$r_2 = 0$	
ordering for apps using MEV taxes			
L1 block builder		$r_1 = r_2 = 0$	
with revert protection			
L ₂ sequencer with	$r_1 = r_2 = 0$		
revert protection			
L ₂ sequencer with revert	$r_1 = r_2 = 0$		
protection for apps using MEV taxes			

Table 2. Revert Penalty Parameters for Various Settings

of r2. Hence, applications are incentivized to capture all but a negligible portion of the priority fee, and thus $r_2 = 0$.

We make the following assumption to avoid trivialities:⁷

Assumption 2.1 Assume that the value exceeds the base fee, i.e., $V > g$.

2.2 Strategy Spaces and Payoffs

Payoffs Under Pure Strategies. Under pure strategies, agent i has strategy space $\mathcal{B} = \emptyset \cup [0,\infty)$ and chooses an action $b_i \in \mathcal{B}$. Given a strategy profile $b =$ $(b_i)_{i\in[N]}$, the payoff of agent i, denoted u_i , is

$$
u_i(b_i|b_{-i}) = \begin{cases} (V - g - b_i)\mathbb{P}(w = i|b) \\ -(r_1g + r_2b_i)(1 - \mathbb{P}(w = i|b)) & \text{if } b_i \ge 0, \\ 0 & \text{if } b_i = \varnothing, \end{cases}
$$

where $\mathbb{P}(w = i | b)$ denotes the probability of agent i winning under the strategy profile b, i.e.,

$$
\mathbb{P}(w = i | b) = \frac{\mathbb{1}\{b_i = b_M\}}{\sum_{j \neq i} \mathbb{1}\{b_j = b_i\}}.
$$

where $b_M \triangleq \max\{b_i : b_i \geq 0\}$, i.e., the highest participating bidder wins, with ties broken at random. The first term in the payoff for participating bidders

⁷ This is because, if $V \leq g$, then the utility an agent receives when winning the action cannot be positive, even if the priority fee is zero, since the value does not exceed the base fee. Thus, all agents abstaining is a dominant strategy equilibrium.

captures the case when agent i has the highest bid; their payoff is the net value gained, $(V - g - b_i)$, scaled by the probability of winning. The second term captures the payment in the case when agent i does not win the auction.

Payoffs Under Mixed Strategies. We now allow agents to probabilistically randomize between the actions of abstaining from the auction and bidding a continuum of possible values. Specifically, agent i now chooses $\beta_i \equiv (p_i, F_i)$ where $p_i \in [0,1]$ is the probability of abstaining $(b_i = \emptyset)$, and F_i is a continuous cdf supported on $b_i \geq 0$ specifying the distribution that agent i bids according to, conditional on agent i choosing to participate in the auction. Given a strategy profile $\beta = (\beta_i)_{i \in [N]}$, the expected payoff \bar{u}_i of agent i conditional on realizing b_i and assuming that agents' actions are chosen independently, is

$$
u_i(b_i|\beta_{-i}) = \begin{cases} (V - g - b_i)\mathbb{P}(w = i|b_i, \beta_{-i}) \\ -(r_1g + r_2b_i)(1 - \mathbb{P}(w = i|b_i, \beta_{-i})) & \text{if } b_i \ge 0, \\ 0 & \text{if } b_i = \varnothing, \end{cases}
$$
 (1)

where $\mathbb{P}(w=i|b_i, \beta_{-i})$ is the probability that agent i wins conditional on realizing b_i and the other agent's strategies. For the assumptions above, we have

$$
\mathbb{P}(w = i | b_i, \beta_{-i}) = \prod_{j \neq i} (p_j + (1 - p_j) F_j(b_i)),
$$
\n(2)

noting that ties occur with probability zero under continuous distributions. The expected utility \bar{u}_i for agent i over their random choice of action b_i is then

$$
\bar{u}_i(\beta_i, \beta_{-i}) \triangleq \mathbb{E}\left[u_i(b_i|\beta_{-i})\right] = (1 - p_i) \int_0^\infty u_i(b|\beta_{-i}) dF_i(b). \tag{3}
$$

3 Equilibrium Analysis

3.1 Pure Strategies

A pure strategy profile b^* as defined in the previous section is a Nash equilibrium in pure strategies if no agent can unilaterally deviate to increase their payoff, i.e for all $i \in [N]$, we have $u_i(b_i^* | b_{-i}^*) \geq u_i(b_i | b_{-i}^*)$ for any $b_i \in \mathcal{B}$.

Theorem 3.1. If $r_1 = r_2 = 0$, and consider a pure strategy profile b. Assume the bids are ordered, so that $b_1 \leq b_2 \leq \cdots \leq b_N$, with the possibility of abstaining represented by bids less than zero. Then b is a Nash equilibrium if and only if $b_{N-1} = b_N = V - g.$

Intuitively, when there is no cost incurred upon losing the auction, the agents have no disincentive associated with large bids, resulting in an equilibrium when at least two agents bid the breakeven bid, i.e., the highest possible amount yielding a nonnegative utility, which is the value of the arbitrage opportunity less the base gas fee.

Theorem 3.2. If at least one of r_1 and r_2 is nonzero, then there does not exist a Nash equilibrium in pure strategies.

Conversely, when agents incur any cost upon losing the auction, whether on the base or priority gas fee, there is no Nash equilibrium in pure strategies. The revert cost penalty results in a situation where at least one agent can benefit from unilaterally deviating given any pure strategy profile. Having characterized the pure-strategy Nash equilibrium and lack thereof, we now look at mixed-strategy equilibria.

3.2 Mixed Strategies

A mixed strategy profile β^* as defined in the previous section is a Nash equilibrium if $\bar{u}_i(\beta_i^*, \beta_{-i}^*) \geq \bar{u}_i(\beta_i, \beta_{-i}^*)$ for all agents i and for any other mixed strategy β_i^* . For tractability, we focus on solving for *symmetric equilibria*, i.e., mixed-strategy Nash equilibria where all agents' strategies β_i are identical.

Theorem 3.3. If at least one of r_1 and r_2 is nonzero, then the unique symmetric mixed-strategy equilibrium is given by

$$
p_i^* = p^* \triangleq \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{1}{N-1}},
$$

$$
F_i^* = F^*(b) \triangleq \frac{1}{1 - p^*} \left(\left(\frac{r_1 g + r_2 b}{V - g - b + r_1 g + r_2 b}\right)^{\frac{1}{N-1}} - p^* \right),
$$

for bids $b \in [0, V - g]$ and for each $i \in [N]$, and the expected payoff of every agent is zero.

Fig. 1. Equilibrium CDF for Priority Fee Bids. We set $V = 10$, $g = 1$, $N = 20$, $r_2 = 0.1$ for the left plot, and $r_1 = 0.1$ for the right plot.

We remark that this equilibrium is unique among symmetric mixed-strategy Nash equilibria, but there may exist non-symmetric equilibria. A complete characterization of all possible equilibria of the auction is outside the scope of this paper, and we focus on the symmetric equilibrium for the rest of the paper.

In order for a mixed-strategy Nash equilibrium to arise, arbitrageurs must be indifferent over all possible actions supported by the mixed strategy. Thus, when the equilibrium assigns a positive probability mass to abstaining from the auction, an action which yields zero payoff, it follows that the equilibrium expected payoff of each arbitrageur is zero. It turns out that under the equilibrium in Theorem [3.3,](#page-8-0) equilibrium expected payoff for arbitrageurs is zero even in cases where they always choose to participate.

When $r_1 > 0$, the symmetric equilibrium strategy is characterized by a nonzero probability of abstention and a non-degenerate continuous CDF specifying the distribution from which to draw the priority gas fee bid. In the special case of $r_1 = 0$, all arbitrageurs participate with probability one. Figures [1](#page-8-1) and [2](#page-9-0) plot the equilibrium CDF and abstention probability for various configurations of parameters. In Appendix [B,](#page-20-0) we discuss comparative statics for the the equilibrium CDF and the abstention probability.

Fig. 2. Equilibrium CDF for Priority Fee Bids (left) and Abstention Probability (right). We set $V = 10$, $g = 1$, and $r_1 = r_2 = 0.1$ for the left plot.

We note that Theorem [3.3](#page-8-0) is consistent with Theorem [3.1.](#page-7-0) When $r_1 = r_2 = 0$, plugging these values into the formula in Theorem [3.3](#page-8-0) yield $p^* = 0$ and $F^*(b) = 0$ for all $b \in [0, V - g)$, assuming that $g \lt V$, with $F^*(V - g)$ being undefined. Compare this with the pure strategy equilibrium given in Theorem [3.1,](#page-7-0) which corresponds to a symmetric mixed-strategy Nash equilibrium given by $p^* = 0$, $F^*(b) = 0$ for all $b \in [0, V - g)$, and $F^*(V - g) = 1$. Indeed, one can show that $F^*(b)$ converges pointwise to the function $\mathbb{1}{b = V - g}$ as $(r_1, r_2) \to (0, 0)$.

4 Implications for Sequencer Design

4.1 Auction Revenue and Market Efficiency

In this section, we examine the comparative statistics of equilibrium quantities that are relevant to sequencer design, starting with auction revenue.

Auction Revenue. Conditional on a realization of the mixed strategy, the auctioneer takes the base gas and priority fee bid of the winner (if one exists), and r_1 times the base gas fee plus r_2 times the priority fee bid by the remaining participating arbitrageurs. Note that the revenue can be split into two terms: one capturing the revenue from the base gas fee, and the other capturing revenue from the priority gas fee bids. The expected revenue over all possible realizations along with its decomposition into base and priority components is characterized by the following theorem.

Theorem 4.1. If at least one of r_1 and r_2 is nonzero, under the symmetric mixed-strategy Nash equilibrium:

– The expected revenue from the auction is

$$
\mathbb{E}[\text{Revenue}] = \left(1 - (p^*)^N\right)V = \left(1 - \left(\frac{r_1g}{V - g + r_1g}\right)^{\frac{N}{N-1}}\right)V.
$$

- Expected revenue decreases in r_1 , does not depend on N when $r_1 = 0$, decreases in N when $r_1 \neq 0$, and does not depend on r_2 .
- Expected revenue can be decomposed into components representing revenue from base gas fees and priority gas fees given by

$$
\mathbb{E}[\text{BaseRevenue}] = (1 - (p^*)^N) g + ((1 - p^*)N - (1 - (p^*)^N)) r_1 g,
$$

$$
\mathbb{E}[\text{PriorityRevenue}] = (1 - (p^*)^N) (V - g) - ((1 - p^*)N - (1 - (p^*)^N)) r_1 g.
$$

 $-I f r_1 \neq 0$, then as the number of arbitrageurs N tends to infinity, the expected revenues converges to a finite limit, with

$$
\lim_{N \to \infty} \mathbb{E}[\text{Revenue}] = \frac{V(V - g)}{V - g + r_1 g},
$$
\n
$$
\lim_{N \to \infty} \mathbb{E}[\text{BaseRevenue}] = \frac{(V - g)(g - r_1 g)}{V - g + r_1 g} - r_1 g \log \left(1 + \frac{V - g}{r_1 g}\right),
$$
\n
$$
\lim_{N \to \infty} \mathbb{E}[\text{PriorityRevenue}] = (V - g) + r_1 g \log \left(1 + \frac{V - g}{r_1 g}\right).
$$

The intuition behind the expression for total revenue is straightforward. Since the arbitrageurs earn a combined expected payoff of zero in equilibrium and the winning arbitrageur extracts a value of V , the total payments made to the sequencer should also equal V , as long as least one arbitrageur participates. We

then take a simple expectation over the events "a value extraction of V occurs" and "no value extraction occurs" which have probabilities $1 - (p^*)^N$ and $(p^*)^N$, respectively, for the result. This phenomena is known as "rent dissipation" [\[7\]](#page-14-9): any value that is realized entirely goes to the sequencer.

Theorem [4.1](#page-10-0) shows that expected revenue decreases in r_1 , the revert penalty rate on the base gas fee. This implies that holding all other auction parameters equal, the revenue-maximizing choice of r_1 in expectation is zero, corresponding to a sequencer with full RP the base gas fee. Furthermore, expected revenue is constant in N under full RP while it decreases in N for nonzero r_1 , highlighting additional losses when full RP is not implemented.

Interestingly, expected revenue does not depend on $r₂$, the revert penalty rate for priority gas fees. The intuition behind this is that value extraction only depends on participation, which is independent of $r₂$. Another interpretation is that r_2 does not matter for the "marginal bidder" who participates but bids a priority fee of zero. Consequently, sequencers and applications can adjust $r₂$ without affecting the expected total payments collected from participants.

This is relevant to considering MEV taxes imposed by applications, which not only change the distribution of the non-base portion of the fee (causing a share of it to go to the application, rather than the sequencer), but also affect r_2 (since MEV taxes are only paid when the transaction succeeds, while a portion of priority fees are paid even on revert). Since r_2 does not affect total revenue, this helps justify the assumption we made in Section [2.1](#page-5-0) that an application will parameterize its MEV taxes as high as possible in order to maximize the revenue that it earns.

The expression for expected auction revenue coming from base gas fees is obtained by first conditioning on the number of participating arbitrageurs k . Given k participants, the sequencer will earn r_1g from $k-1$ of them and g from the remaining one. Subtracting this from the total revenue thus gives the component coming from priority gas fee bids. Notably, these components also do not depend on r_2 .

As previously mentioned, the probability of a value extraction occurring lowers as N increases, stemming from the disincentive to participate brought on by a more competitive environment, but this probability converges to a finite limit as N tends to infinity. This provides a minimum guarantee on expected revenue for any number of arbitrageurs.

Market Efficiency. Revenue is deeply connected with market efficiency. Recall that the probability that value from the arbitrage opportunity is extracted is $1-(p^*)^N$. When this quantity is high, on-chain arbitrages are frequent, thereby improving market efficiency. On the other hand, when this quantity is low, onchain arbitrages are less likely to occur, leaving arbitrage opportunities unexploited and generating zero value for the auction. It is straightforward to see that r_1 has the same directional impact on market efficiency (when measured by the probability that a value extraction occurs) as expected auction revenue described in Theorem [4.1,](#page-10-0) so more revert protection (lower r_1) implies a higher market efficiency.

4.2 Blockspace and Mempool Usage

We proceed to analyze blockspace and memory pool usage. If an arbitrageur decides to participate, they submit a transaction to the mempool. When transactions are not fully revert-protected, they will still appear on-chain, which is prevented by full revert protection. The following theorem characterizes these quantities in equilibrium.

Theorem 4.2. Under the symmetric mixed-strategy Nash equilibrium:

– The expected number of submitted transactions is

$$
\mathbb{E}[\text{SubmittedTXs}] = (1 - p^*)N = \left(1 - \left(\frac{r_1g}{V - g + r_1g}\right)^{\frac{1}{N-1}}\right)N.
$$

- Expected transactions submitted decreases in r_1 and does not depend on r_2 .
- If $r_1 \neq 0$, then as the number of arbitrageurs N tends to infinity, expected transactions submitted converge to a finite limit, with

$$
\lim_{N \to \infty} \mathbb{E}[\textsf{SubmittedTXs}] = \log \left(1 + \frac{V - g}{r_1 g} \right).
$$

Theorem [4.2](#page-12-0) implies that a higher revert penalty rate r_1 on the base gas fee has a dampening effect on arbitrageur participation, with the expected number of submitted transactions decreasing in r_1 . Similarly to expected revenue, expected transactions submitted is constant in r_2 since submission only depends on the participation probability.

Under full revert protection, all arbitrageurs will participate, so as N tends to infinity, so does the number of submitted transactions. When transactions are not fully protected, expected transactions submitted are bounded irrespective of the number of arbitrageurs. In terms of blockspace efficiency, full revert protection is more efficient, particularly as V grows large, since only the winning arbitrageur's transaction will appear on the block as opposed to all participating arbitrageur's transactions when full RP is not in place. However, under full RP, there may be arbitrarily many transactions submitted to the mempool as N grows large, compared to a bounded number of submitted transactions otherwise.

4.3 Discussion: Full Revert Protection

Fig. 3. Expected Auction Revenue (left) and Transactions Submitted (right). We set $V = 10$ and $q = 1$.

We summarize the impact of full revert protection on arbitrage dynamics within a blockchain by analyzing and contrasting the expected number of participants and auction revenue in equilibrium under two conditions: one where $r_1 \neq 0$, and the other where $r_1 = 0$, i.e., full revert protection (for the base gas fee component). Figure [3](#page-13-0) shows these quantities for various values of r_1 and N.

With full revert protection, the sequencer is always able to capture the full value of an arbitrage opportunity, resulting in an expected auction revenue of V . In this scenario, since there is no deterrent to submitting bids, all arbitrageurs are incentivized to bid up to the breakeven point. As the number of arbitrageurs increases, this can present a spam risk to the sequencer. However, because reverts do not occur on-chain, at most one transaction appears on-chain when an arbitrage opportunity arises, or none otherwise. Thus, the primary challenge for the sequencer under a full revert protection is managing spam effectively.

When revert protection is not in place, the number of participants in an auction asymptotically approaches a finite number as the number of arbitrageurs grows indefinitely. Conversely, auction revenue is reduced by a factor of at most $(1-(p^*)^N)$ compared to the case of full revert protection. While full revert protection can enhance auction revenue and improve blockspace efficiency, it introduces the risk of increased spamming as the number of arbitrageurs grows. The central trade-off, therefore, lies between these benefits and the elevated risk of spam due to potentially unlimited bids as more arbitrageurs enter the system.

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A Proofs

A.1 Proof of Theorem [3.1.](#page-7-0)

Define $b^* \triangleq V - g$ to be the breakeven bid, and let b be an ordered pure strategy equilibrium.

Consider the following cases:

- $b_N > b^*$: In this case, the highest bidder b_N will incur a loss, as they are bidding more than the break-even bid b^* . This bidder would be better off abstaining, contradicting the fact that b is an equilibrium.
- $b_N < b^*$: Here, the highest bidder b_N bids less than the break-even bid, allowing them to make a profit if they win. However, another bidder can deviate by bidding slightly above b_N but below b^* , ensuring they win with certainty and still make a profit. Thus, the original set of bids cannot be in equilibrium, as there is an incentive to deviate.
- $b_N = b^*$ and $b_{N-1} < b^*$: In this case, the top bidder is bidding exactly at the break-even level. However, the top bidder can increase their profit by reducing their bid to slightly above b_{N-1} , as they would still win but at a lower bid cost. Hence, this setup also cannot be in equilibrium.

Conversely, suppose that $b_N = b_{N-1} = b^*$. Then, no matter the bids of other players, all players have zero utility, and cannot increase their utility through any deviation. Hence, this is an equilibrium.

A.2 Proof of Theorem [3.2.](#page-7-1)

Suppose that b is an pure strategy equilibrium, and assume the bids are ordered, so that $b_1 \leq b_2 \leq \cdots \leq b_N$, with the possibility of abstaining represented by bids less than zero. Define the breakeven bid as $b^* \triangleq V - g$.

Consider the following cases:

- $b_N > b^*$: In this case, the highest bidder b_N will incur a loss, as they are bidding more than the break-even bid b^* . This bidder would be better off abstaining, contradicting the fact that b is an equilibrium.
- $b_N < b^*$: Here, the highest bidder b_N bids less than the break-even bid, allowing them to make a profit if they win. However, another bidder can deviate by bidding slightly above b_N but below b^* , ensuring they win with certainty and still make a profit. Thus, the original set of bids cannot be in equilibrium, as there is an incentive to deviate.
- $b_N = b^*$ and $b_{N-1} < b^*$: In this case, the top bidder is bidding exactly at the break-even level. However, the top bidder can increase their profit by reducing their bid to slightly above b_{N-1} , as they would still win but at a lower bid cost. Hence, this setup also cannot be in equilibrium.
- $b_N = b_{N-1} = b^*$: If the top two bidders both bid the breakeven value $b^* > 0$, then they will make no profit when they win, and, because at least one of r_1, r_2 is positive, they will have strictly negative utility if they lose. Then, at least one player bidding b^* will have strictly negative expected utility and would be better off abstaining. Therefore, this is not a sustainable equilibrium either.

A.3 Proof of Theorem [3.3](#page-8-0)

We analyze a symmetric mixed strategy Nash equilibrium where arbitrageurs randomize their bids over a range of possible values. Let p represent the probability that an arbitrageur abstains from bidding, and for those who bid, let $F(b)$ be the cumulative distribution function (CDF) representing the probability that the bid is strictly less than b.

Suppose an agent bids $b \geq 0$. From [\(1\)](#page-7-2)–[\(3\)](#page-7-3), the expected probability of winning the auction is given by

$$
(p+(1-p)F(b))^{N-1},
$$

and thus the expected payoff is

$$
(p + (1-p)F(b))^{N-1} (V - g - b) - (1 - (p + (1-p)F(b))^{N-1}) (r_1g + r_2b).
$$

In a mixed strategy equilibrium, an arbitrageur must be indifferent between bidding and not bidding. That is, the expected payoff from bidding any b must be zero, the same as the expected payoff from abstaining. This leads to the following indifference condition

$$
(p + (1 - p)F(b))^{N-1} (V - g - b) = (1 - (p + (1 - p)F(b))^{N-1}) (g \cdot r_1 + b \cdot r_2).
$$

Solving for $F(b)$, we have

$$
F(b) = \frac{1}{1-p} \left(\left(\frac{r_1 g + r_2 b}{V - g - b + r_1 g + r_2 b} \right)^{\frac{1}{N-1}} - p \right).
$$

The boundary condition $F(0) = 0$ yields

$$
F(0) = \frac{1}{1-p} \left(\left(\frac{r_1 g}{V - g - r_1 g} \right)^{\frac{1}{N-1}} - p \right) = 0.
$$

Solving for p , we have that

$$
p = \left(\frac{r_1g}{V - g + r_1g}\right)^{\frac{1}{N-1}}.
$$

Note that $F(V - g) = 1$, so that agents only bid in the range $b \in [0, V - g]$.

Now, assume that the agents all adopt the strategy (p, F) . We have established that any individual agent is indifferent between abstaining, and any bid $b \in [0, V - g]$ — all of these actions result in zero expected payoff. Since bidding $b > V - g$ leads to a negative expected payoff, such bids can be excluded. Therefore, the agents have no incentive to deviate, and we have established a Nash equilibrium.

A.4 Proof of Theorem [4.1](#page-10-0)

Expected Revenue. Each arbitrageur's expected utility can be decomposed into a "value extracted" and "payment" component:

$$
\bar{u}_i(\beta_i^*,\beta_{-i}^*)=\mathbb{E}[\mathsf{ValueExtracted}_i]-\mathbb{E}[\mathsf{Payment}_i].
$$

As each arbitrageur's expected utility is zero in equilibrium, summing all of them up yields

$$
0 = \mathbb{E}\left[\sum_i \text{ValueExtracted}_i\right] - \mathbb{E}\left[\sum_i \text{Payment}_i\right] \\ = \mathbb{E}[\text{TotalValueExtracted}] - \mathbb{E}[\text{Revenue}],
$$

so it follows that

$$
\mathbb{E}[\mathsf{Revenue}] = \mathbb{E}[\mathsf{TotalValueExtracted}].
$$

With probability $(p^*)^N$, no arbitrageurs participate, so no value is extracted. With probability $1 - (p^*)^N$, at least one arbitrageur participates. In this case, only the winning arbitrageur will receive a value of V , with the others receiving zero value. Thus

$$
\mathbb{E}[\text{Revenue}] = (p^*)^N \cdot 0 + (1 - (p^*)^N)V = (1 - (p^*)^N)V.
$$

Comparative Statics of Expected Revenue. Note that

$$
\mathbb{E}[\text{Revenue}] = 1 - (p^*)^N = 1 - \left(\frac{r_1g}{V-g+r_1g}\right)^{\frac{N}{N-1}}.
$$

– To show that $\mathbb{E}[\text{Revenue}]$ decreases in r_1 , note that

$$
\frac{r_1g}{V-g+r_1g}
$$

increases in r_1 .

– To show that $\mathbb{E}[\text{Revenue}]$ decreases in N, note that

$$
\frac{\partial}{\partial N} \left(\frac{r_1 g}{V - g + r_1 g} \right)^{\frac{N}{N - 1}} = -\frac{1}{(N - 1)^2} \left(\frac{r_1 g}{V - g + r_1 g} \right)^{\frac{N}{N - 1}} \log \frac{r_1 g}{V - g + r_1 g}
$$

which is nonnegative since

$$
\frac{r_1g}{V-g+r_1g} < 1.
$$

– It is straightforward to see that $\mathbb{E}[\text{Revenue}]$ does not depend on r_2 , and when $r_1 = 0$, we have $\mathbb{E}[\text{Revenue}] = V$ which is constant in r_2 .

.

Decomposition of Expected Revenue. To derive the base gas fee revenue, let N_P be a random variable for the number of agents that participate. Conditional on $N_P = k$ for $k \ge 1$, note that the winner pays the full base fee g while the $(k-1)$ losers pay the revert cost of r_1g . Then

$$
\mathbb{E}[\text{BaseRevenue}] = \sum_{k=1}^{N} \mathbb{P}(N_P = k) \cdot (g + (k-1)r_1g)
$$

= $r_1 g \sum_{k=1}^{N} k \cdot \mathbb{P}(N_P = k) + (1 - r_1)g \sum_{k=1}^{K} \mathbb{P}(N_P = k)$
= $N(1 - p^*)r_1 g + (1 - (p^*)^N)(1 - r_1)g.$

The revenue from priority fees is then given by

E[PriorityRevenue] = E[Revenue] − E[BaseRevenue]

$$
= (1 - (p^*)^N)V - (N(1 - p^*)r_1g + (1 - (p^*)^N)(1 - r_1)g)
$$

Limiting Values of Revenue Components. Note that

$$
1 - (p^*)^N = 1 - \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{N}{N-1}}.
$$

As $N \to \infty$, note that $N/(N-1) \to 1$, so

$$
\lim_{N \to \infty} 1 - \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{N}{N - 1}} = 1 - \frac{r_1 g}{V - g + r_1 g} = \frac{V - g}{V - g + r_1 g}.
$$

Similarly, note that

$$
(1 - p^*)N = \left(1 - \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{1}{N - 1}}\right)N.
$$

As $N \to \infty$, the exponent $1/(N-1)$ tends to zero, so we have

$$
\lim_{N \to \infty} \left(1 - \left(\frac{r_1 g}{V - g + r_1 g} \right)^{\frac{1}{N - 1}} \right) N = \lim_{N \to \infty} \left(-\frac{1}{N - 1} \log \frac{r_1 g}{V - g + r_1 g} \right) N
$$

$$
= \log \frac{V - g + r_1 g}{r_1 g}.
$$

Plugging these expressions into the formulas for expected revenue, expected base revenue, and expected priority revenue yields the result.

A.5 Proof of Theorem [4.2](#page-12-0)

Expected Transactions Submitted. For $i \in [N]$, let Z_i be an indicator random variable for arbitrageur i participating in the auction. Then

$$
\mathbb{E}[\textsf{SubmittedTXs}] = \mathbb{E}\left[\sum_{i \in [N]} Z_i\right] = \sum_{i \in [N]} \mathbb{P}(Z_i = 1) = N(1 - p^*).
$$

Comparative Statics of Expected Transactions Submitted. Note that

$$
\mathbb{E}[\text{SubmittedTXs}] = (1 - p^*)N = \left(1 - \frac{r_1 g}{V - g + r_1 g}\right)^{\frac{1}{N - 1}} N
$$

– To show that $\mathbb{E}[\textsf{SubmittedTXs}]$ decreases in r_1 , note that

$$
\frac{r_1g}{V-g+r_1g}
$$

increases in r_1 .

– It is straightforward to see that $\mathbb{E}[\mathsf{SubmittedTXs}]$ does not depend on $r_2.$

Limiting Number of Expected Transactions Submitted. The expression follows from the proof of Theorem [4.1.](#page-10-0)

B Comparative Statics of the Equilibrium

We examine how the equilibrium strategy changes with respect to the auction parameters N, V, g, r_1 and r_2 . Specifically, we analyze the parameters' effect on the equilibrium abstention probability p^* and the equilibrium priority gas fee bidding distribution F^* via the expected bid $\mathbb{E}B_i^*$ where $B_i^* \sim F^*$ as a summary statistic.

Theorem B.1. The equilibrium probability of abstention from the auction, p^* ,

- increases in N, g and r_1 ;
- $-$ decreases in V.

The expected priority gas fee bid of an arbitrageur conditional on participation in the auction, $\mathbb{E}B_i^*$ where $B_i^* \sim F^*$,

- $-$ increases in V ;
- decreases in N, r_1 , and r_2 .

These results are fairly intuitive. As the number of arbitrageurs increases, each arbitrageur has more competitors, reducing incentives to participate and bid higher in expectation due to the penalty associated with not winning. When the value of the arbitrage opportunity increases, the additional value incentivizes arbitrageurs to participate and submit higher expected bids. The effect of the base gas fee, g, on participation probability follows a similar intuition, but that on the expected priority fee bid is not characterizable in general without further specification of parameters.

As the revert penalty rate on the base gas fee increases, arbitrageurs are less inclined to participate and bid higher on average due tot he increase in revert costs. The revert penalty rate on the priority fee has a similar effect on the bidding distribution, but notably does not affect the participation probability at all. This is because a marginal arbitrageur who participates but bids zero priority fee is indifferent about the revert cost on priority fees.

Proof of Theorem [B.1.](#page-20-1) Note that

$$
\frac{\partial p^*}{\partial N} = -\frac{1}{(N-1)^2} \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{1}{N-1}} \log \frac{r_1 g}{V - g + r_1 g} \ge 0
$$

$$
\frac{\partial p^*}{\partial g} = \frac{1}{N-1} \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{1}{N-1}} \frac{V}{g(V - g + r_1 g)} \ge 0
$$

$$
\frac{\partial p^*}{\partial r_1} = \frac{1}{N-1} \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{1}{N-1}} \frac{V - g}{r_1 (V - g + r_1 g)} \ge 0
$$

$$
\frac{\partial p^*}{\partial V} = -\frac{1}{N-1} \left(\frac{r_1 g}{V - g + r_1 g}\right)^{\frac{1}{N-1}} \frac{1}{V - g + r_1 g} \le 0
$$

where p^* is the equilibrium abstention probability given by

$$
p^* = \left(\frac{r_1g}{V-g+r_1g}\right)^{\frac{1}{N-1}}.
$$

Note that

$$
\frac{\partial F^*}{\partial N} = -\frac{\left(\frac{r_1g + r_2b}{V - g - b + r_1g + r_2b}\right)^{\frac{1}{N-1}}}{(1 - p)(N - 1)^2} \log\left(\frac{r_1g + r_2b}{V - g - b + r_1g + r_2b}\right) \ge 0
$$

$$
\frac{\partial F^*}{\partial V} = -\frac{\left(\frac{r_1g + r_2b}{V - g - b + r_1g + r_2b}\right)^{\frac{1}{N-1}}}{(1 - p)(N - 1)(V - g - b + r_1g + r_2b)} \le 0
$$

$$
\frac{\partial F^*}{\partial r_1} = \frac{g(V - g - b)\left(\frac{r_1g + r_2b}{V - g - b + r_1g + r_2b}\right)^{\frac{1}{N-1}}}{(1 - p)(N - 1)(r_1g + r_2b)(V - g - b + r_1g + r_2b)} \ge 0
$$

$$
\frac{\partial F^*}{\partial r_2} = \frac{b(V - g - b)\left(\frac{r_1g + r_2b}{V - g - b + r_1g + r_2b}\right)^{\frac{1}{N-1}}}{(1 - p)(N - 1)(r_1g + r_2b)(V - g - b + r_1g + r_2b)} \ge 0
$$

where F^* is the equilibrium CDF given by

$$
\frac{1}{1-p^*}\left(\left(\frac{r_1g+r_2b}{V-g-b+r_1g+r_2b}\right)^{\frac{1}{N-1}} - p^* \right).
$$

If $\partial F^*/\partial \theta \geq 0$ where θ is some parameter of interest, then the CDF increases pointwise for all $b \in [0, V - g]$. Then for any θ_1, θ_2 such that $\theta_1 < \theta_2$, letting $B_{\theta_1}^* \sim F_{\theta_1}^*$ and $B_{\theta_2}^* \sim F_{\theta_2}^*$, it follows that $B_{\theta_1}^*$ stochastically dominates $B_{\theta_2}^*$ in the first order, so $\mathbb{E}B_{\theta_1}^* > \mathbb{E}B_{\theta_2}^*$.

If $\partial F^*/\partial \theta \leq 0$ where θ is some parameter of interest, then the CDF decreases pointwise for all $b \in [0, V - g]$. Then for any θ_1, θ_2 such that $\theta_1 < \theta_2$, letting $B_{\theta_1}^* \sim F_{\theta_1}^*$ and $B_{\theta_2}^* \sim F_{\theta_2}^*$, it follows that $B_{\theta_2}^*$ stochastically dominates $B_{\theta_1}^*$ in the first order, so $\mathbb{E}B_{\theta_1}^* \leq \mathbb{E}B_{\theta_2}^*$.